

# Primordial Generation of Magnetic Fields

Jitesh R. Bhatt, Arun Kumar Pandey

**Abstract** We reexamine generation of the primordial magnetic fields, at temperature  $T > 80\text{TeV}$ , by applying a consistent kinetic theory framework which is suitably modified to take the quantum anomaly into account. The modified kinetic equation can reproduce the known quantum field theoretic results upto the leading orders. We show that our results qualitatively matches with the earlier results obtained using heuristic arguments. The modified kinetic theory can give the instabilities responsible for generation of the magnetic field due to chiral imbalance in two distinct regimes: a) when the collisions play a dominant role and b) when the primordial plasma can be regarded as collisionless. We argue that the instability developing in the collisional regime can dominate over the instability in the collisionless regime.

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## 1 Introduction

Observations suggest that we live in a magnetized Universe: Magnetic fields are present from stellar scales to the intergalactic scales. However, it is yet not clear that how these magnetic field arises. One of the possible thought is that their origin is due to some process in the early universe like inflation[1] or phase transitions[2] etc. It may also possible that the magnetic fields may not have any primordial origin but they may be generated during a gravitational collapse[3]. Currently the generation and dynamics of the primordial magnetic field is one of the most intriguing problem in cosmology [4].

In recent time, there has been great interests in generation of primordial fields by quantum anomaly[5]. A chiral imbalance of the leptons can occur in the very early universe, due to some electroweak(EW) anomalous processes like parity violating decays of massive particles, just before EW scale[6]. It has been shown that, the presence of chiral imbalance creates instability in the the hot matter[7]. At temperatures  $T > 80 \text{ TeV} (\sim T_R)$ , the chirality flipping processes are highly suppressed and hence asymmetry between the right-handed and left-handed particle is preserved. In the regime  $T > T_R$ , the EW symmetry is unbroken and the generated field will be  $U(1)$  hypermagnetic(hypercharge) fields[8]. After EW phase transition, these hypermagnetic field will be converted into the electromagnetic field.

Recently there has been growing interests in including the parity violating effects into a kinetic theory formalism. It was found that the chiral

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anomaly can be incorporated in a kinetic theory framework by including the Berry curvature correction[9]. The resulting theory can reproduce the triangle anomaly and provide the descriptions for the chiral magnetic and chiral vorticity effects[10]. In this work we apply the Berry curvature modified kinetic theory to the chiral plasma for the study of generation of primordial magnetic field. In this work, we discuss the generation and evolution of magnetic field in two regions: a) where collision play a dominant role i.e.  $\nu_c \gg \omega \gg k$  where  $\nu_c$  is the collision frequency,  $\omega$  and  $k$  respectively denote the typical frequency and wave-number for the perturbations. b) And in the collisionless regime  $k \gg \omega \gg \nu_c$ . The instabilities can grow in these two regimes at the expense of the chiral-imbalance and surprising the maximum growth rates for both regimes occur at the similar length scales. In this work, we analyze the characteristics of these instabilities within the framework of the modified kinetic theory and discuss the relationship between our results and the earlier results. The manuscript is organized into two sections. In first section we discussed the generation of primordial magnetic field incorporating Berry correction. Second section contains results and discussion.

## 2 Generation of the primordial magnetic field

In this work we solve a coupled system of the Maxwell equations and the modified kinetic theory for relativistic particles in the spatially flat Friedmann-Lemaître-Robertson-Walker universe. Conformal flatness of this space-time insures that equations in the conformal space have same forms as those of the flat space-time[11]. A conformal metric can be written as:

$$ds^2 = a^2(\eta)(-d\eta^2 + dx^2 + dy^2 + dz^2). \quad (1)$$

where conformal time  $\eta$  is related with proper time  $t$  by  $\eta = \int dt/a^2(t)$ . The quantities like (hyper)-electric  $\mathbf{E}$ , (hyper)-magnetic  $\mathbf{B}$  and the current density  $\mathbf{J}$  measured by the comoving observer are related with quantities in the conformally flat space quantities through scale factor as:  $\mathbf{E} = a^2\mathcal{E}$ ,  $\mathbf{B} = a^2\mathcal{B}$  and  $\mathbf{J} = a^2\mathcal{J}$ .

By inclusion of Berry correction to the kinetic theory, the distribution function as well as kinetic equations modifies. The modified kinetic equation under relaxation time approximation, at the first order in perturbation is:

$$\left( \frac{\partial}{\partial \eta} + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} \right) f_i + \left( e^i \mathbf{E} + e^i (\mathbf{v} \times \mathbf{B}) - \frac{\partial \epsilon_p^i}{\partial \mathbf{r}} \right) \cdot \frac{\partial f_i}{\partial \mathbf{p}} = \left( \frac{\partial f_i}{\partial \eta} \right)_{coll.}, \quad (2)$$

The index  $i$ , in above equation represents different species of the leptons.  $\Omega_{\mathbf{p}}$  comes for Berry curvature and defined by  $\Omega_{\mathbf{p}} = \pm \mathbf{p}/(2p^3)$ . Positive and negative signs comes for right handed fermions and for left handed fermions respectively.  $\epsilon_{\mathbf{p}}^i$  is defined as  $\epsilon_{\mathbf{p}}^i = p(1 - e^i \mathbf{B} \cdot \Omega_{\mathbf{p}}^i)$  with  $p = |\mathbf{p}|$ . Depending on

the species charge  $e$ , energy of the particles  $\epsilon_p$ , Berry curvature  $\mathbf{\Omega}_p$  and form of the distribution function  $f$  changes. In the absence of Berry correction i.e.  $\mathbf{\Omega}_p = 0$ , above equation reduces to the Vlasov equation when the collision term on the right hand side of equation(2) is absent. With inclusion of Berry curvature, current density also modifies as:

$$\mathbf{J}_i = -e^i \int \frac{d^3p}{(2\pi)^3} \left[ \epsilon_p^i \frac{\partial f_i}{\partial \mathbf{p}} + e^i (\mathbf{\Omega}_p^i \cdot \frac{\partial f_i}{\partial \mathbf{p}}) \epsilon_p^i \mathbf{B} + \epsilon_p \mathbf{\Omega}_p^i \times \frac{\partial f_i}{\partial \mathbf{r}} \right] \quad (3)$$

Above temperature  $T > T_R$ , the chiral plasma remains in thermal equilibrium. And masses of the plasma particles can be ignored. So plasma in equilibrium can be considered to be in homogeneous and isotropic state. So distribution function for different species in thermal equilibrium can be written as  $f_{0i}(p) = [\exp(\frac{p-\mu_i}{T}) + 1]^{-1}$ . Now let us suppose that,  $\delta f_i$  is fluctuations in the distribution functions of the particle species around it's equilibrium distribution, then perturbed distribution function can be written as:  $f_i(\mathbf{r}, \mathbf{p}, \eta) = f_{0i}(p) + \delta f_i(\mathbf{r}, \mathbf{p}, \eta)$ . Now one can obtain the total current  $\mathbf{J}$  by adding the contribution from all the species of the particle  $\mathbf{J}_a$ . Total current comes out:

$$\begin{aligned} \mathbf{J}_k = & -m_D^2 \int \frac{d\Omega}{4\pi} \frac{\mathbf{v}(\mathbf{v} \cdot \mathbf{E}_k)}{i(\mathbf{k} \cdot \mathbf{v} - \omega - i\nu_c)} - \frac{c_D^2}{2} \int \frac{d\Omega}{4\pi} \left( \frac{\mathbf{v}(\mathbf{v} \cdot \mathbf{B}_k)(\mathbf{k} \cdot \mathbf{v}) - (\mathbf{v} \times \mathbf{k})(\mathbf{v} \cdot \mathbf{E}_k)}{(\mathbf{k} \cdot \mathbf{v} - \omega - i\nu_c)} + \mathbf{B}_k \right) \\ & - \frac{ig_D^2}{4} \int \frac{d\Omega}{4\pi} \frac{(\mathbf{v} \times \mathbf{k})(\mathbf{v} \cdot \mathbf{B}_k)(\mathbf{k} \cdot \mathbf{v})}{(\mathbf{k} \cdot \mathbf{v} - \omega - i\nu_c)} - \frac{h_D^2}{2} \int \frac{d\Omega}{4\pi} \{ \mathbf{B}_k - \mathbf{v}(\mathbf{v} \cdot \mathbf{B}_k) \} \end{aligned} \quad (4)$$

To get the expression (4) from eq.(3), we solved eq.(2) to get distribution function. Here  $\Omega$  represent angular integrals. In eq.(4), we have defined  $m_D^2 = e^2 \int \frac{p^2 dp}{2\pi^2} \frac{df_0}{dp}$ ,  $c_D^2 = e^2 \int \frac{p dp}{2\pi^2} \frac{df_0}{dp}$ ,  $g_D^2 = e^2 \int \frac{dp}{2\pi^2} \frac{df_0}{dp}$  &  $h_D^2 = e^2 \int \frac{dp}{2\pi^2} f_0$  and  $f_0 = \sum_a f_{0i}$ . Plasma with chirality imbalance are known to have instabilities that can generate magnetic fields in two different regimes: (i) for the case when  $k \ll \omega \ll \nu_c$  [12] and (ii) in the quasi-static limit i.e.  $\omega \ll k$  and  $\nu_c = 0$  [7]. In this section we analyze how the magnetic fields evolve in the plasma due to these instabilities, within the modified kinetic theory frame work. Using Maxwell's equation and current expression given above, one can get diffusivity equation for chiral plasma in the regime  $k \ll \omega \ll \nu_c$  as:

$$\frac{\partial \mathbf{B}_k}{\partial \eta} + \left( \frac{3\nu_c}{4\pi m_d^2} \right) k^2 \mathbf{B}_k - i \left( \frac{\alpha \Delta \mu}{\pi m_D^2} \right) \left( \mathbf{k} \times \frac{\partial \mathbf{B}_k}{\partial \eta} \right) + i \frac{4\alpha \nu_c \Delta \mu}{\pi m_D^2} (\mathbf{k} \times \mathbf{B}_k) = 0. \quad (5)$$

We can solve this equation without a loss of generality by considering the propagation vector  $\mathbf{k}$  in  $z$ -direction and the magnetic field having components perpendicular to  $z$ -axis. After defining two new variables:  $B_k = (B_k^1 + iB_k^2)$  and  $\tilde{B}_k' = (B_k^1 - iB_k^2)$  one can rewrite eq.(5) as:

$$\frac{\partial \tilde{B}_k}{\partial \eta} + \frac{3\nu_c}{4\pi m_d^2} k \left[ \frac{k - \frac{16\alpha\Delta\mu}{3}}{(1 + \frac{\alpha\Delta\mu k}{\pi m_D^2})} \right] \tilde{B}_k = 0, \quad (6)$$

$$\frac{\partial \tilde{B}'_k}{\partial \eta} + \frac{3\nu_c}{4\pi m_d^2} k \left[ \frac{k + \frac{16\alpha\Delta\mu}{3}}{(1 - \frac{\alpha\Delta\mu k}{\pi m_D^2})} \right] \tilde{B}'_k = 0. \quad (7)$$

Another regime where chiral imbalance instability can occur is in the quasi-static limit when  $\omega \ll k$  and  $\nu_c = 0$  [7]. In this case the diffusivity equation comes out:

$$\frac{\partial \mathbf{B}_k}{\partial \eta} + \frac{k^2}{4\pi\sigma_1} \mathbf{B}_k - i \frac{\alpha T \delta}{\pi\sigma_1} (\mathbf{k} \times \mathbf{B}_k) = 0 \quad (8)$$

where  $\sigma_1 = \pi m_D^2/2k$ . One can get decoupled equation as in the case of  $\nu_c > \omega > k$ . Which is:

$$\frac{\partial \tilde{B}_k}{\partial \eta} + \left[ \frac{k^2 - \frac{4\alpha\Delta\mu k}{3}}{\frac{\pi m_D^2}{2k}} \right] \tilde{B}_k = 0, \quad (9)$$

$$\frac{\partial \tilde{B}'_k}{\partial \eta} + \left[ \frac{k^2 + \frac{4\alpha\Delta\mu k}{3}}{\frac{\pi m_D^2}{2k}} \right] \tilde{B}'_k = 0. \quad (10)$$

### 3 Results and discussion

Equations (6-7, 9-10) describe the evolution of the primordial magnetic field due to quantum anomaly for both the collisional and collisionless regimes. For the case when  $\nu_c \gg \omega \gg k$ , modes described by eq.(6), becomes unstable when  $k < (16/3)\alpha\Delta\mu$ . The instability has the maximum growth rate of  $\gamma_1 \sim \frac{16}{3\pi} \frac{T^2 \delta^2}{m_D^2} \nu_c$  for  $k_{max}^1 \sim \frac{8\alpha T \delta}{3}$ . Eq.(7) can also give the instability if the denominator of the second term on the right hand side become negative. But this possibility is ruled out for the present case. For  $\tilde{B}'_k$  in eq.(7), the modes are damped if the condition  $\left( \frac{\alpha T \delta}{\pi m_D^2} \right) k < 1$  is satisfied. This condition is always satisfied since  $\alpha \ll 1$ ,  $k/m_D < 1$  and  $T/m_D \sim O(1)$ . Thus eq. (7) can not give the unstable modes. The value of  $k_{max}^1$  is of same order of magnitude as the one given in Ref.[5]. But in the collisionless regime, the modes of  $\tilde{B}_k$ , as seen from eq.(9), become unstable for  $k < 4\alpha\Delta\mu/3$ . The instability has maximum growth rate  $\gamma_2 \sim \frac{\alpha^3}{2\pi} \left( \frac{T\delta}{m_D} \right)^2 (T\delta)$  at  $k_{max}^2 \sim \frac{8\alpha T \delta}{9}$ . Eq.(10) describes the purely damping modes. It has been found that, instability in the two regime has almost same length scale for which their modes start growing. So it is interesting to ask, if instabilities in the two regime occurs almost at the same length scale, then which one will win. For this we compare the maximum growing rates in the two regimes and we

found that  $\frac{\gamma_1}{\gamma_2} \sim 10(\alpha\delta)^{-1}$ . Clearly for  $\delta \ll 1$ , it is the collisional plasma whose modes will grow faster than the modes of the collisionless plasma. However for the chiral plasma, where  $\delta \gg 1$  the situation may be reversed. The magnetic field will continue to grow at the cost of the chiral charge. This can be seen from the anomaly equation which at  $T > 80$  TeV gives  $n_L - n_R + 2\alpha\mathcal{H} = \text{constant}$ , where,  $n_{L,R} = \frac{\mu_{L,R}T^2}{6}$  and  $\mathcal{H}$  is the magnetic helicity.

In conclusion, we have applied the kinetic theory with the Berry curvature correction to study origin of the primordial magnetic field due to anomaly. We have incorporated the effect of collisions using the relaxation time approximation. Further we have shown that the instability that can be present in a collisionless chiral plasma may not grow as fast as one found in presence of collision (in Ref.[12]) when  $\delta \ll 1$ .

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